Strut and Tie Topology for Reinforced Concrete Elements

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Abstract

Strut and Tie Model (STM) is a lower-bound design method for reinforced concrete members. It is usually preferred for non-standard structural elements for which internal forces are not well-defined. In recent years, studies and applications on STM have increased considerably. Topology optimization methods are generally used to find struts and ties. Once locations and orientations of reinforcements are available, the design procedure is rather easy to apply, even for complex geometries. However, finding the STM topology is not a straight-forward procedure. In this study, STM topology is obtained automatically by applying a simple algorithm. The algorithm involves coupled computations carried out on an analog lattice model and a continuum model. The stiffness of lattice elements is iteratively modified based on internal force distribution. Similarly, elasticity modulus of continuum is modified based on principal stress distribution. During this process, redundant lattice elements and continuum elements disappear. Lattice model and continuum model affect each other to arrive at an optimized STM topology. Solutions obtained with the developed algorithm is compared with available computational results from literature and good agreements are observed. Furthermore, STM topology for a non-trivial geometry is presented to demonstrate the applicability of the methodology.

Keywords: *Strut and tie modeling, topology optimization, finite element method, continuum elements, lattice model.*

1 Introduction

The foundations of the strut and tie modeling were laid with the lattice-beam simulation method proposed by Ritter and Mörsch in the early years of the 20th century, which was later followed by the studies carried out by Leonhardt, Rüsch and Kupfer. A generalized form of this method was developed by Schlaich et al. (1987) and named the strut and tie approach. In the numerical analysis, it was necessary to define regions that exhibit different types of behavior. Cook and Mitchell (1988) defined regions of discontinuity that arise due to sudden changes in section dimensions or loading. In the study, the use of strut and tie modeling in the analysis and calculation of short cantilevers, short cantilever beams, hollow beams and deep beams were studied. The compression (struts) and tension (ties) bars were used to estimate the bearing capacities of the D (discontinuity) regions within acceptable limits. Likewise, Schlaich et al. (1991) focused on the formulation of strut and tie modeling to determine the boundaries and capacities of D regions.

Strut and tie modeling is similar to the truss analogy used in structural analysis. Although Mörsch (1909) was the first researcher to propose this concept, it has been used extensively in the last two decades to analyze strut and tie models in structural elements such as deep beams (Chen et al., 2019), hollow beams (Ali et al., 2001), corbels (Saeed et al., 2009) and shear walls (Yun et al., 2008).

Studies have shown that the structural elements can be modeled not only with truss elements but also with plane finite elements. Blaauwendraad and Hoogenboom (1996) stated that it is possible to analyze many structural elements such as beams, walls, protruding and cantilever beams as two-dimensional plane finite elements for which both loading and support responses remain in the same plane. With the optimization of the plane elements, the strut and tie models in the structure elements can be created manually using engineering intuition and experience. Zhong et al. (2017) defined the dapped beam element with plane elements and proposed a methodology based on evolutionary structural optimization (ESO). They placed strut and tie elements by using the topology and strain energy concentration regions created by volume reduction. Xia et. get. (2020), developed a methodology that utilizes two variables (volume fraction and filter radius). Volume fraction affects the number of finite elements and their width. The filter radius was used to smoothen the topology optimization during deletion of finite elements.

Topology optimizations studied in the literature generally make volume reduction according to the energy variation for both lattice or plane finite elements. Although these methods have been shown to work sufficiently well for mold casting (Sato et. al. 2017) or 3D printing of such as machine parts (Meng et. al. 2020), they are not adequate or robust in determining strut and tie topologies for reinforced concrete structural elements.

In this study, a novel optimization method that couples lattice and continuum models is proposed to arrive at an optimized strut and tie topology for two dimensional reinforced concrete members. In this approach, lattice and continuum material properties are functions of internal force for the lattice model and principal stresses for the continuum model. Preliminary results show that the method is robust and works with the same set of parameters for problems with quite different geometries.

2 Proposed Approach for Topology Optimization

A topology optimization approach was developed in the context of this study. The novelty of the approach is that both lattice and continuum model results are used to vary material properties (axial stiffness for lattice elements and elasticity modulus for continuum elements). This approach is shown to overcome difficulties that may arise when only lattice or continuum model is used for topology optimization.

The proposed topology optimization has been tested on different structural members and different problems in the literature. The approach is presented by using the beam example studied by Liang et al. (2000). In that study, beams with four different L/D (length over depth) ratios were studied with an applied point load P = 1200 kN, Young's modulus of concrete E_c = 28567 MPa and Poisson's ratio ν = 0.15. The beam with L/D = 2 is used to present the developed approach (Figure 1).

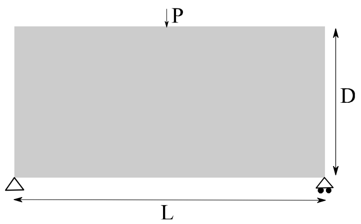
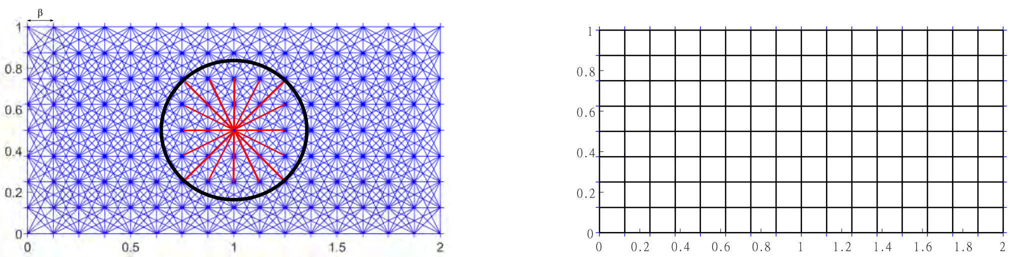


Figure 1. Load and support condition for L/D=2.

First finite element meshes were obtained for both lattice and continuum models. In order to create the lattice mesh, each node is connected to nodes within a certain distance δ (Figure 2-a). It is assumed that this distance is proportional to the minimum mesh size. For the L/D=2 model, if each node is connected to all other nodes, one iteration takes 263 seconds. When the lattice model is created by connecting the points within the δ radius circle (δ =2.8 β), one iteration takes 41 seconds. Both lattice models give similar results. Therefore, δ was taken as 2.8 β in all of the simulations. The continuum system was modeled using 4-noded quadrilateral finite elements (Figure 2-b).



(a) (b)

Figure 2. (a) Lattice mesh with 1464 elements

(b) Continuum mesh with 128 elements

Both continuum and lattice systems were analyzed for the same load and support conditions. In the first simulation, unit modulus of elasticity is assumed for all lattice elements. For the continuum model, the modulus of elasticity was taken as the elasticity modulus of concrete. Lattice and continuum deformation fields are then compared (Figure 3).

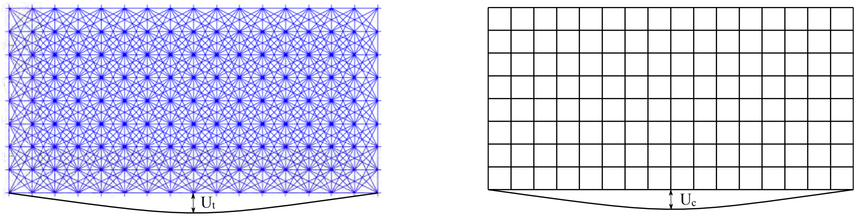


Figure 3. Deformation as a result of the first analysis

Assuming that the two representative displacements (which are U_t and U_c) are equal to each other, the truss elasticity modulus (E_t) is calculated from the following expression

$$E_t = \left(\frac{U_t}{U_c} \right) \tag{1}$$

Representative displacements could be maximum displacements as is the case here or could be chosen as average displacement for problems with irregular geometries. According to the new elasticity value for lattice elements, internal forces are recalculated, and maximum and minimum internal normal forces are determined:

$$\begin{aligned} N_{\max} &= \max(N_i) \\ N_{\min} &= \min(N_i) \end{aligned} \tag{2}$$

where N_i, N_{max} and N_{min} are internal normal force for lattice element i, the maximum and minimum internal forces for the lattice system. Continuum model is also solved to obtain the state of stress for each finite element. In addition, principal stresses are calculated:

$$\begin{aligned} \sigma_m^{\ 1} &= \max\left(\left|\sigma_j^{\ 1}\right|\right) \\ \sigma_m^{\ 2} &= \max\left(\left|\sigma_j^{\ 2}\right|\right) \end{aligned} \tag{3}$$

where σ_m^1, σ_m^2, σ_j^1 and σ_j^2 are maximum first principal stress, maximum second principal stress, first principal stress of the jth continuum element and second principal stress of the jth continuum element, respectively.

Using the principal stresses, σ_m^1 and σ_m^2, a new modulus of elasticity (E_x, E_y) is assigned to each finite element of the continuum system. With this step, while the modulus of elasticity of the load-bearing finite elements (E_c) is increased, the modulus of elasticity of the non-load-bearing finite elements is decreased (minimum value E_m is taken as 10^{-6} in the example calculations)

$$\begin{aligned} E_x &= \left(\frac{\sigma_j^{\ 1}}{\sigma_{\max}^{\ 1}} \right)^2 E_c + E_m \\ E_y &= \left(\frac{\sigma_j^{\ 2}}{\sigma_{\max}^{\ 2}} \right)^2 E_c + E_m \\ E_{c,i} &= \max(E_x, E_y) \end{aligned} \tag{4}$$

Lattice elements passing through each continuum element are determined (Figure 4). The lattice with the maximum modulus of elasticity ($E_{max,t}$) is found.

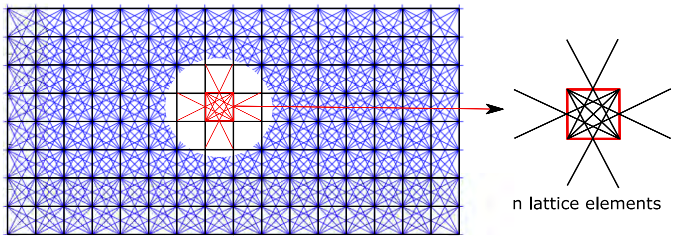


Figure 4. Intersection of truss and continuum systems

The new elasticity modulus of each continuum element ($E_{c,j}$) is calculated using the following expression

$$E_{c,j} = \left(\frac{E_{c,j}}{E_c} + \frac{E_{\max,t}}{E_t} \right) E_c \tag{5}$$

Here, E_c is the original modulus of elasticity of concrete which is kept constant in the analysis. The two terms within the parenthesis indicate that the updated modulus of elasticity of continuum element j is a function of maximum lattice element elasticity and $E_{c,j}$ which is a function of principal stress ratio of element j.

The continuum elements cut by a lattice element are found and average elasticity modulus of these elements are calculated ($E_{avg,p}$) (Figure 5).

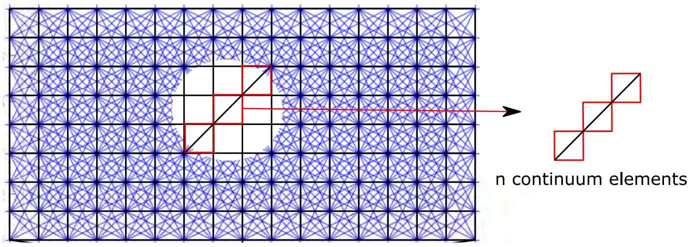


Figure 5. Intersection of continuum and truss systems

The updated modulus of elasticity is calculated for each lattice element from

$$E_{T,i} = \left(\frac{E_{t,j}}{E_t} + \frac{E_{avg,p}}{E_c} \right) E_t \tag{6}$$

Elasticity modules of lattice elements are recalculated according to internal forces and deletion of lattice elements with an elasticity module less than $E_{threshold}$ from the system.

$$\begin{aligned} E_{T,i} &= E_{T,i} A \left(\frac{q_i}{q_{\max}} \right)^2 \qquad if \qquad q_i > 0 \\ E_{T,i} &= E_{T,i} A \left(\frac{q_i}{q_{\min}} \right)^2 \qquad if \qquad q_i < 0 \end{aligned} \tag{7}$$

where $E_{T,i}$ and A are updated elasticity modulus of each continuum element and cross-sectional area of lattice element, respectively. Iterations continue until the change in the stiffness of lattice and membrane elements is less than a tolerance (this value was taken as %1 in the simulations presented below).

3 Results

The results obtained using the algorithm given above area are first compared with the results from literature. Liang et al. (2000) studied strut and tie topology for beams with different L/D ratios. As seen in Figure 6, for L/D=5 extra struts and ties are observed in the solution of Liang et al. (2000). It should be noted that struts and ties were drawn manually by Liang et al. (2000) by using the force flow lines in the continuum system. The same problem was solved for L/D=2 and L/D=5 with the proposed method, and the results are shown in Figure 6. The mesh used for the beam with L/D=2 had 153 nodes, 1464 lattice elements and 128 continuum elements whereas the mesh used for the beam with L/D=5 had 369 nodes, 3696 lattice elements and 320 continuum elements.

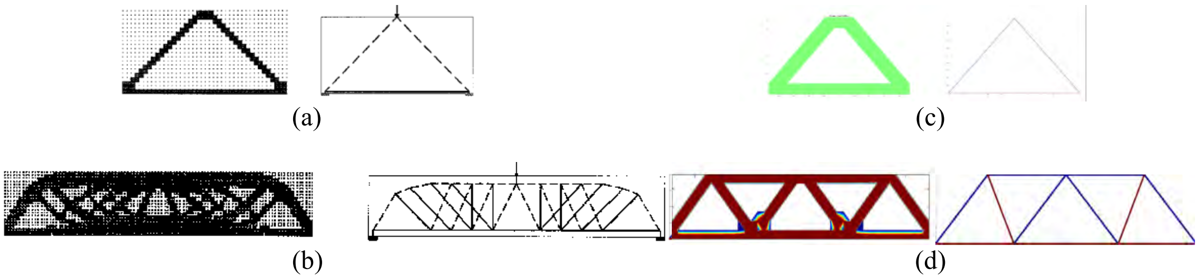


Figure 6. Optimal topologies and lattice models showing transition from deep beams to slender beams: (a) L/D=2; (b) L/D=5 (Liang et al. (2000)); (c), Topology optimization results for L/D=2; (d) L/D=5.

Figure 6 shows that strut and tie models can be obtained with the proposed topology optimization method. For L/D=2, results are identical. However, for L/D=5, results are different. It should be noted that the topology obtained with the proposed topology optimization is preferable for the design process. Indeed, topology obtained by Liang et al. (2000) should be simplified before the strut and tie design process can be applied.

Gaynor et al. (2013) studied a beam with an irregular shape including a hole shown in Figure 7. They used a method based on ESO and revised the stiffness matrix according to the displacements in the continuum elements. In this study, the modulus of elasticity is 24.9 GPa and ν = 0.2 for concrete. The mesh used in the study had 420 nodal points, 4006 lattice elements and 352 continuum elements. When the same problem was analyzed with the proposed optimization method, a similar strut and tie topology was obtained with some minor variations as shown in Figure 7.

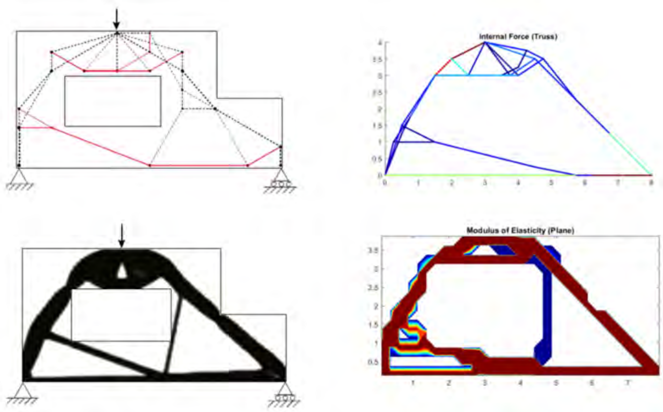


Figure 7. (a) Gaynor et al. (2013), (b) Proposed topology optimization method

4 Conclusion

In this study, a novel optimization method that couples lattice and continuum model was presented. The objective was to gather the advantages of both models while, at the same time, eliminating the weaknesses. The axial stiffness for the lattice model and elasticity modulus for the continuum model are not only nontrivial functions of force and stress distribution but also functions of each other. The preliminary numerical results show that the obtained strut and tie topologies allows for the easy design of reinforced concrete structural members with non-standard geometry for which internal force-based design process is known to fail. Further numerical simulations, as well as an experimental program, are planned to be carried out for the validation of the obtained strut and tie topologies.

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